Exploring Gauge Theories with Gravity

1. D-Branes
2. The Maldacena Conjecture
3. Compact Spaces and Orbifolds
4. Fractional and Wrapped D-Branes

Low-Energy Limit of Strings

- Closed strings (Type II) gives a theory of gravity.
- Open strings (Type I) gives a Yang-Mills (gauge) theory, *and* a theory of gravity.

D-Branes, Open String Viewpoint:

D-branes are endpoints for open strings.

- If we put Dirichlet boundary conditions in some directions for the endpoints of an open string, the string will be constrained to end on a subspace.
- (Dirichlet boundary conditions must be considered because of T-duality, which exchanges Neumann and Dirichlet boundary conditions.)
- Since the open string theory gives a YM theory in the low-energy limit, there will be YM theories “living on the branes”.

D-Branes, SUGRA Viewpoint:

D-Branes are objects in supergravity with Ramond-Ramond-charge

- In the *low-energy limit*, D-Branes are solutions of supergravity (Type IIA and IIB) corresponding to massive objects with Ramond-Ramond-charge.
- This means that a Dp-brane is a source for the $p+1$-form field $C_{(p+1)}$.
- Since type II SUGRA is the low-energy limit of type II string theory, we find that D-branes also exist in *closed* string theory!

D3-Brane Solution in SUGRA

Action for type IIB supergravity:

$$S = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} \left[R - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{2\cdot 5!}(\mathrm{d}\mathcal{C}_{(4)})^2 \right]$$

A solution of the eqs. of motion is

$$\begin{aligned} ds^2 &= [H(r)]^{\frac{1}{2}}(-dt^2 + dx^2 + dy^2 + dz^2) \\ &\quad + [H(r)]^{-\frac{1}{2}}(dr^2 + r^2 d\Omega_5^2) \\ \mathcal{C}_{0123} &= [H(r)]^{-1} \\ \phi(x) &= \text{const.} \end{aligned}$$

where

$$H = 1 + \frac{R^4}{r^4}$$

$$R^4 = 4\pi g_s {\alpha'}^2 N$$

This solution may be shown to be BPS (it conserves half of the supersymmetry) and corresponds to a D3-brane in string theory.

$\alpha' \to 0$: Open String Viewpoint

The total action may be separated into

$$S = S_{\text{open}} + S_{\text{closed}} + S_{\text{int}}$$

- S_{open}: Action for open strings (on the brane)
- S_{closed}: Action for closed strings (in the bulk)
- S_{int}: Interaction between open and closed

In the limit $\alpha' \to 0$, $S_{\text{int}} \to 0$.

In the low-energy-limit, the closed string part S_{closed} becomes a free supergravity theory, and the open string part S_{open} becomes pure $\mathcal{N} = 4$ $U(N)$ Yang-Mills theory.

Perturbative Yang-Mills results are valid when $g_{\text{YM}}^2 N \sim g_s N \ll 1$.

$\alpha' \to 0$: SUGRA Viewpoint

Far from the D-brane, space is nearly flat.

In the *near horizon limit*,

$$ds^2 \approx \frac{r^2}{R^2}(-dt^2 + dx^2 + dy^2 + dz^2) + R^2(\frac{dr^2}{r^2} + d\Omega_5^2)$$

which is the geometry of $AdS_5 \times S^5$.

In the limit $\alpha' \to 0$, these regions separate.

We are left with free gravity for $r \to \infty$ and $AdS_5 \times S^5$ near the horizon.

The supergravity approximation is valid when R is large, which means that $g_s N \gg 1$.

AdS-CFT Correspondence

In the low-energy limit with $\alpha' \to 0$:

- From open strings, we got

$$\mathcal{N} = 4 \quad U(N) \quad \text{YM} \qquad + \qquad \text{free gravity}$$

- From supergravity, we got

$$AdS_5 \times S^5 \qquad + \qquad \text{free gravity}$$

Therefore:

$$\boxed{\textbf{SUGRA on } AdS_5 \times S^5 \;\equiv\; \mathcal{N} = 4 \; U(N) \; \textbf{YM}}$$

This is the **AdS-CFT Correspondence**.

Since SUGRA is valid in the strong coupling limit of the YM theory, we expect to be able to get non-perturbative results of the YM theory from SUGRA.

The Maldacena Conjecture

Maldacena[a] conjectured that this duality can be taken to the string theory level, i.e.

Type IIB String Theory on $AdS_5 \times S^5$

$$\equiv$$

$\mathcal{N} = 4$ $U(N)$ **Yang-Mills in** $d = 3 + 1$

Similar conjectures may be made by starting from other brane configurations.

Is it possible to make a connection between a more physical gauge theory and gravity?

---

[a] J. Maldacena, *Adv. Theor. Math. Phys.* **2** (1998), 231 (hep-th/9711200)

Probing

Properties of the Yang-Mills theory may be investigated through the dual gravity theory with a technique known as *probing*:

- A D-brane is placed in the background of N parallel D-branes.
- The β-function of the theory may be found by inspecting the coupling constant of the YM theory on the probe as the probe moves transversly to the brane.
- Result from D3-brane:

$$\beta = 0$$

Compactification

To compactify a space means to "roll up" some of the dimensions:

Compactification *reduces the number of supersymmetries* in a string theory. Therefore, we might find a more "phenomenologically" interesting duality by compactifying the space containing the D-brane.

Compactification

The properties of the gauge theory will also depend on how the D-brane is embedded into the compact space.

One or more directions parallel to the brane may be compactified.

There are well known conditions on the embedding if we want to conserve some supersymmetries.

Orbifolds

Dealing with strings in compact spaces is difficult. However, some simplified versions of such spaces exist:

An orbifold is made by taking a smooth space M (e.g. flat space $\mathbb{R}^n$), and identifying points transforming into each other under some group Γ. The resulting space is an orbifold if it has singular points and is denoted by M/Γ.

One simple example: $M = \mathbb{R}^4$, $\Gamma = \mathbb{Z}_2$, where the action of the group is to take $(x_1, x_2, x_3, x_4) \mapsto (-x_1, -x_2, -x_3, -x_4)$.

The singular points are points that are invariant under the group action, in this case only the origin $(0,0,0,0) \mapsto (0,0,0,0)$.

Orbifolds

An orbifold of the type $\mathbb{R}^4/\Gamma$ can be shown to be a limit of a smooth space called an ALE (Asymptotically Locally Euclidean) space. In this sort of space, the singular point is replaced by a compact manifold.

In the limit where the compact manifold has zero volume, the orbifold is recovered.

For the case of $\mathbb{R}^4/\mathbb{Z}_2$, the compact manifold is a sphere S^2.

D-Branes on Orbifolds

D-Branes localized on the singular point of an orbifold have special properties. Such a D-brane may be a *fractional* D-brane.

A fractional D-brane has a fraction of the charge of a regular D-brane. On an orbifold $\mathbb{R}^4/\mathbb{Z}_n$, the fractional D-branes will have charge $\frac{Q}{n}$ with respect to the regular brane charge Q.

A fractional D-brane may be seen as a D-brane wrapping the vanishing cycle of an ALE space.

E.g. a fractional D3 brane may be seen as a D5 brane with two dimensions wrapped.

D-Branes on Orbifolds

A supergravity solution for a fractional D-brane in an orbifold may be solved by viewing the D-brane as being wrapped on a vanishing cycle.

Result for D3-Brane on $\mathbb{R}^4/\mathbb{Z}_2$ corresponds to pure $\mathcal{N} = 2$ $U(N)$ Yang-Mills theory. However, some problems show up in this case:

- No non-perturbative results
- The tension of the probe vanishes at a point called the "enhançon"

These results are believed to be connected, and the enhançon is believed to be the point where the perturbative analysis breaks down.

D-Branes on Orbifolds

Result from probing the fractional brane[a]:

$$g_{\rm YM}^2(\mu) = g_0^2 \left(1 + \frac{N g_0^2}{4\pi^2} \log \mu \right)^{-1}$$

Result from probing the fractional D2 brane[b]:

$$g_{\rm YM}^2(\mu) = g_0^2 \left(1 - \frac{N g_0^2}{4\pi \mu} \right)^{-1}$$

These results are exactly the *perturbative* results from $U(N)$ Yang-Mills theory in $3+1$ and $2+1$ dimensions.

---

[a] M. Bertolini et al., *JHEP* 0102:014 (2001) (hep-th/0011077)

[b] Di Vecchia, Imeroni, Lozano, Enger, "coming soon"...

Wrapped D-Branes

It is also possible to find solutions corresponding to branes wrapping cycles of *finite volume* in ALE-like spaces.

Unfortunately, the same types of problems:

- Only perturbative results
- Enhançon

seems to occur in this case.

Conclusion/Outlook

- The Maldacena conjecture is a duality between closed string theory and Yang-Mills theory.
- The Maldacena conjecture is still a topic of active research, after 4 years.
- By understanding the enhançon-related problems, we *may* learn something about QCD.
- Are there other interesting Maldacena-like dualities?

References

[1] J. Maldacena, *Adv. Theor. Math. Phys.* **2** (1998), 231 (hep-th/9711200)
[2] M. Bertolini et al., *JHEP* 0102:014 (2001) (hep-th/0011077)
[3] Di Vecchia, Imeroni, Lozano, Enger, “coming soon”. . .

Reviews
C. Johnson, hep-th/0007170
P. Di Vecchia and A. Liccardo, hep-th/9912161
O. Aharony et al., hep-th/9905111