The Problem with the Cosmological Constant in Quantum Field Theory

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1 Introduction

Near the beginning of this century, there were two major break-throughs in physics: The general theory of relativity (GR) and the theory of quantum mechanics (QM). The theory of Relativity, which is usually regarded as the work of one man, Albert Einstein, describes the motion of the planets, galaxies and indeed the evolution of the whole universe as well as we are currently able to measure. Unlike relativity, which has remain unchanged since 1916, QM has been refined by a great number of physicists throughout the century, and the theories of Quantum Electrodynamics (QED) and Quantum Chromodynamics (QCD) now describe with an incredible precision the motion of sub-atomic particles. It is ironic that while these theories were discovered at roughly the same time, and while both are extremely accurate in each of these fields, they seem almost impossible to combine into one single theory, and nobody has yet come up with a satisfactory description of the laws of nature that is able to predict both the motion of large scale and small scale objects at the same time.

One of the places where this problem manifests itself is with the problem of the cosmological constant. The cosmological constant was introduced by Einstein into GR in order to create a solution of the equations of GR which described a static universe. At the moment there was no evidence of any large scale movement in the universe, and a static solution seemed to be a necessary condition for any equation to correctly describe the universe. In Einstein's words, "The most important fact that we draw from experience is that the relative velocities of the stars are very small as compared with the velocity of light."[1] In addition, the introduction of the cosmological constant ensured that an empty universe satisfied Mach's principle[2]. With a cosmological constant λ, Einstein's field equations are

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu} - \lambda g^{\mu\nu} = -8\pi G T^{\mu\nu}. \tag{1}$$

When the red shift of distant galaxies was observed by Slipher in the period 1910–1924, it was realized that this condition was not necessary anymore, and it seemed natural to take the constant out of the equations again. Einstein later described the cosmological constant as his biggest blunder. However, the fact that the universe is expanding rather that static is naturally not a proof that the cosmological constant is exactly zero, only that it is significantly smaller than expected by Einstein.

In later years, there seem to be a significant number of observations, most recently studies of type Ia supernovae[3, 4], indicating a non-zero cosmological constant. Since the cosmological constant does not seem to be "required" from a theoretical point of view, many physicists do not like the idea of putting it into Einstein's equations "by hand." The question we will consider in this essay, is whether quantum physics will ever be able to *predict* the value of the cosmological constant.

All equations in this essay use the convention $\hbar = c = 1$.

2 The Vacuum Energy Problem

At first, quantum field theory (QFT) looks promising regarding this question. In quantum field theory, the zero point energy of the fields apparently gives a non-zero energy for vacuum. Since vacuum is Lorentz invariant, this energy fulfills

$$T^{\mu\nu} = -\rho_V g^{\mu\nu}, \tag{2}$$

and therefore the vacuum energy appears in Einstein's field equations as an effective cosmological constant,

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu} = -8\pi G T^{\mu\nu} = -8\pi G(T^{\mu\nu}_{\text{eff}} - \rho_V g^{\mu\nu}), \tag{3}$$

which may be transformed into equation (1) by letting $\lambda = 8\pi G\rho_V$. Thus, even if we do not include a cosmological constant by hand into the equations, we arrive at a non-zero cosmological constant that might explain the observed one.

Unfortunately, is isn't quite this easy. First of all, the vacuum energy formally diverges. The zero point energy of a quantum harmonic oscillator is $\frac{1}{2}\hbar\omega$. For a quantum field, $\omega = \sqrt{k^2 + m^2}$, where k is the wave vector. Integrated over all possible wave vectors, this diverges catastrophically! This is normally not considered a problem in quantum field theory, since the energy of vacuum is assumed not to be detectable in any experiment—what you measure is the energy difference of a given state and the vacuum. However, since the energy of vacuum gives, as we have seen, a contribution to the cosmological constant, the result seems catastrophic. Quantum field theory appears to predict an infinite cosmological constant.

It turns out that the divergence in the vacuum energy is not the only infinity that appears in quantum field theory. All amplitudes one tries to calculate to a higher order diverges. Fortunately, quantum field theorists have found a way to circumvent these infinities. The process used is known as *renormalization*. With this process, it is possible to "isolate" the infinities and by relating certain amplitudes, or probabilities, to actual experimental results, one may substitute finite values for the infinite term. Renormalization is not a topic for this essay, but the concept of renormalization is important because it has resulted in a new interpretation of the standard model. The simplest form of renormalization is simply to replace the upper limit on the integral for the vacuum energy with a constant Λ:

$$\rho_V = \int_0^{\Lambda} \frac{4\pi k^2 dk}{2(2\pi)^3} \sqrt{k^2 + m^2}. \tag{4}$$

This immediately leads to a finite result. This Λ was at first considered to be a mathematical peculiarity which should be removed by taking the limit $\Lambda \to \infty$ after all the calculations were done. Since amplitudes (probabilities) for physical processes did not in fact diverge in this limit, this seemed to solve the problem of infinities. However, in recent years the upper limit has been interpreted as an upper limit to the Standard Model itself. For energies above the energy level Λ, one needs to introduce new physics. This physics is unknown to us, and it gives negligible contribution at energy levels much smaller than Λ, but it is reasonable to believe that at some point, new physics needs to be introduced.

This seems to provide a new way to solve the problem of the cosmological constant. Λ would be assumed to have a value such that the integral (4) corresponds to the observed cosmological constant. Of course, we could still have a *bare* cosmological constant in addition to the "dynamical" vacuum energy. The total cosmological term in Einstein's field equations would then include an *effective* cosmological constant $\lambda_{\text{eff}} = \lambda + 8\pi G\rho_V$, where ρ_V is the vacuum energy.

Unfortunately, we know that the standard model is correct at least up to the energy levels of current particle physics experiments, or about 200 GeV. With this (very low) value for Λ, the vacuum energy becomes approximately, by equation (4),

$$\rho_V \approx \frac{\Lambda^4}{16\pi^2} \approx 10^7\text{GeV}. \tag{5}$$

The current observational evidence suggests a cosmological constant of the order of $\lambda \approx 10^{-29} g/cm^3$, which corresponds to $\rho_V \approx 10^{-47}$GeV. We find a mismatch of the theoretical and experimental values of the cosmological constant of about 54 orders of magnitude! Even worse, if we choose to believe that the Standard Model is correct all the way up to the Planck Scale, $\Lambda \approx (8\pi G)^{\frac{1}{2}}$, where relativistic effects are believed to become important, the mismatch grows to 120 orders of magnitude.

One possible solution to this inconsistency would be to include, by hand, a bare cosmological constant into the Lagrangian. This constant would then have to cancel the effect of the vacuum energy with an accuracy of at least 50 decimal places. Such a cancellation does not seem to be a very likely accident!

3 Symmetries

One way of making the cosmological constant smaller than expected from current theory, is an unknown symmetry that forces the vacuum energy to be small. Such results of symmetries are well known in QFT, one example is the small mass of the π-mesons, which comes from the approximate symmetry between up and down quarks. There are several possible symmetries that may force the cosmological constant to be close to zero, three of which we shall present below.

3.1 Supersymmetry

The most studied variant is supersymmetry. Supersymmetry is an extension to the standard model that, in very few words, identifies fermions and bosons as the same type of particles. We can not go into details of the supersymmetry mechanism in this essay, but the formalism includes a set of supersymmetry generators $\{Q_\alpha\}$, where $\alpha = 1, 2$. These operators are formed by combining creation and annihilation operators for fermions and bosons. Since they are generators for supersymmetry, a supersymmetric vacuum state must satisfy

$$Q_\alpha |0\rangle = Q_\alpha^\dagger |0\rangle = 0. \tag{6}$$

Another result of the supersymmetry formalism is that the energy-momentum operator P^μ has a very simple form:

$$(\sigma_\mu)_{\alpha\beta} P^\mu = \{Q_\alpha, Q_\beta^\dagger\}. \tag{7}$$

This implies that the expectation value of the vacuum energy $\langle 0| P^0 |0\rangle$ must be identical to zero.

If this was the whole truth, and since observations seem to indicate that the cosmological constant has a small non-zero value, the observed cosmological constant must come from the *bare* cosmological constant, which must be introduced as a parameter into the theory that can not be deduced from any first principles.

But it is not this simple. We know for sure that supersymmetry is not an observed symmetry of nature. If we still chose to believe that supersymmetry is a fundamental symmetry, we must conclude that supersymmetry is *spontaneously broken*, i.e. that the vacuum of the real world is not supersymmetric. In that case, equation (6) no longer holds, and the vacuum energy is no longer identical to zero.

As optimists, we might expect that the breaking of supersymmetry would then give a value for the vacuum energy that agrees with the observed value. The breaking of supersymmetry introduces a mass difference between pairs of fermions and bosons that belong to the same multiplet in the supersymmetric theory. Since we have not seen any "supersymmetric partners" of any of the known elementary particles, their mass must be greater than the current observational bound, which is about 200 GeV. The energy of the vacuum in the broken theory would be expected to be of the same order of magnitude as this mass difference, so again we end up with a result that contradicts observation with at least 50 orders of magnitude.

3.2 Scale Invariance

At the classical level, the Standard Model can be formulated scale invariantly[5], since it contains only dimensionless coupling constants. However, when quantum corrections are introduced, the scale invariance vanishes. If we were able to keep the scale invariance, both the vacuum energy and the bare cosmological constant Λ would have to disappear.

The main problem is again, as with supersymmetry, that we know that the world is not scale invariant. We have very strong experimental results backing the scale dependence in the quantum field theory of the Standard Model. Thus, the scale invariance would somehow need to be broken.

Stephen Adler[6] has proposed a new kinematic framework called *Generalized Quantum Dynamics* (GQD) to solve the problems relating to combining a fundamental scale invariance with the observed non-invariance. We are not able to discuss the full implementations of this framework, but the major result is that the vacuum energy exactly vanishes on the QFT level if the underlying GQD theory is scale invariant. It is not clear whether it is possible to obtain a bare cosmological constant from these principles.

3.3 Weyl Symmetry

The Weyl transformation is a conformal transformation where the quantum fields are transformed as under a scale transformation. Joan Solà[5] has proposed to extend the Standard Model with a local Weyl symmetry, by introducing new gauge bosons. This leads to a dynamical cosmological "constant", which has a zero expectation value.

Solà's construction does not need or include a Higgs particle, so this could be a candidate theory if there continues to be no experimental evidence for a Higgs particle. However, Weinberg[1] argues that any such theory introducing new fields to "adjust" the vacuum energy will give too many conditions on the fields, so that no solution can be found without fine tuning the parameters. This is not a very nice situation, and Solà does not seem to have developed his theories further recently.

4 Quintessence

The term "Quintessence" is used generically for various exotic effects that may have a contribution to the energy density in the universe similar to that of a cosmological constant. Quintessence is defined to have an equation of state different from known matter (baryons, leptons, radiation and dark matter)[7]. One example of Quintessence might be the new gauge bosons from section 3.3. Another might be macroscopic objects like cosmic strings[7].

There are of course enormous possibilities for introducing new physics that might have such an effect, and since the new type of matter is generally assumed to interact very weakly with ordinary matter (to explain why it has not been detected in experiments), it seems somewhat presumptuous to assume the existence of any particular kind of such matter. Nevertheless, some general models have been studied.

One such model is the so called "tracker fields"[8], which are a form of Quintessence assumed to be slowly evolving by "rolling down" a potential. This model has been introduced to explain the "cosmic coincidence" problem—why should the energy density of Quintessence be so close to the energy density of matter in today's universe?

The "tracker field" Q is assumed to satisfy an equation of motion,

$$\ddot{Q} + 3H\dot{Q} + V'(Q) = 0, \tag{8}$$

where H is the Hubble parameter, and for certain potentials $V(Q)$, this equation allows solutions where the cosmological "constant", which will now be a function of Q, ends up with a value near the density of matter more or less independently of the initial conditions.

There still remains interpretational problems with this method, and it is not clear how it withstands Weinberg's arguments presented in the previous section.

5 Quantum Cosmology and the Anthropic Principle

By applying the concepts of quantum mechanics to the whole universe, the Cosmological Constant could become a dynamical quantity which would not be exactly determined. In this picture, there would be a distribution of values for Λ. Hawking[9] and Coleman[10] have shown that in such a theory, the distribution will have a sharp peak at $\Lambda = 0$. If this is a correct picture, it might seem disturbing that current observations show that the Cosmological Constant does not seem to be exactly zero. Furthermore, a quantum theory of the whole universe leads to a rather overwhelming interpretation problem.

Another proposed solution to the Cosmological Constant problem is the "Anthropic Principle", summarized by Weinberg[1]. This view considers the likeliness of existence of observers to be a criteria for the possible values taken by the Cosmological Constant and other fundamental constants. The argument is that these constants must take values such that planets can form and life establish itself, or else nobody would be here to measure them. In one way, this view may be connected to the Quantum Cosmology theory by interpreting the probability distribution for the universe as the actual existence of an infinite number of "sub-universes", each with a different value for Λ.

Weinberg shows that the Anthropic Principle indeed gives a range of values for the vacuum energy density in the universe near the mass density of matter. He also argues that since the mass density for matter changes with time, the Anthropic Principle might be the only adequate explanation of why the vacuum energy density should be close to the matter energy density.

However, it seems that such an explanation for the value of the Cosmological Constant (and other constants) is a kind of a defeat for physics. It would be satisfactory to explain the problem by finding a more fundamental physical theory rather than resorting to anthropic principles.

6 Conclusion

The problem of the Cosmological Constant and Quantum Field Theory is one of the great unsolved problems in modern physics. For that reason, there have been many proposals for solutions, none of which have so far proved conclusive. It has not been possible in this essay to give more than a brief summary of some of the attempts. More thorough discussions may be found in the reviews by Weinberg[1] and by Carroll and Press[11].

The Anthropic Principle seems to indicate that the range of values allowed for a cosmological constant in a universe where observers can exists are not very large. However, it would still be satisfactory to find a "first principle" predicting the value of λ. At the moment the search continues, since none of the methods presented in this essay have yet proved conclusive.

One important point to make is the strong relation between the more general problem of combining Quantum Field Theory with General Relativity and the problem of the Cosmological Constant. It is impossible to solve the latter problem completely before one has found a satisfactory solution to the former. Therefore, it might be too early to try to solve the Cosmological Constant problem at the moment. On the other hand, if one is able to solve the problem of the Cosmological Constant, its solution might give a hint to the solution of the QFT-GR combination problem.

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