Dynamics of self-sustained activity in random networks with strong synapses

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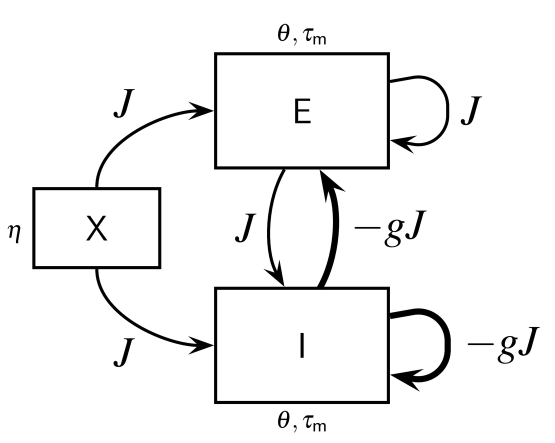
Summary

- Random network with strong current-based synapses display self-sustained activity (SSA).
- Emergence due to concave response function.
- SSA is strongly fluctuating.
- Average lifetime of SSA depends on network parameters, increases with synapse strength.
- Our stochastic model predicts lifetime from correlations between neurons.

Balanced Random Network Model



A	**Network model:** Multi-population random network with fixed in-degrees (Brunel, 2000)
Populations	E (excitatory): LIF neurons (see **B**), size N_E
	I (inhibitory): LIF neurons (see **B**), size N_I
	X (external): Poisson point processes with rate $\eta \nu_\theta$, size $K_E(N_E+N_I)$
Connectivity	EE, IE: Random convergent $K_E \to 1$, excitatory synapses
	EI, II: Random convergent $K_I \to 1$, inhibitory synapses (see **C**)
	EX, IX: Non-overlapping $K_E \to 1$, excitatory synapses (see **C**)
Parameters	Population sizes $N_{\{E,I\}}$, in-degrees $K_{\{E,I\}} = \varepsilon N_{\{E,I\}}$, connectivity ε, relative external drive η

B	**Neuron model:** Leaky integrate-and-fire (LIF) neuron (Lapicque, 1907; Tuckwell, 1988)
Spike emission	Neuron $k \in [1, N_E+N_I]$ fires at all times $\{t_{j_k} \mid V_k(t_{j_k}) = \theta,\ j_k \in \mathbb{N}\}$
Subthreshold dynamics	$\tau_m \dot{V}_k = -V_k + RI_k(t)$ if $\forall j_k : t \notin (t_{j_k}, t_{j_k}+\tau_{ref}]$ Total synaptic input current $I_k(t) = \sum_l \sum_{j_l} i_{kl}(t-t_{j_l})$ (see **C**)
Reset + refractoriness	$V_k(t) = V_{reset}$ if $\forall j_k : t \in (t_{j_k}, t_{j_k}+\tau_{ref}]$
Parameters	Membrane time constant τ_m, membrane resistance R, spike threshold θ, reset potential V_{reset}, refractory period τ_{ref}

C	**Synapse model:** Static current synapse with α-function shaped PSC
PSC kernel	$i_{kl}(t+d) = \begin{cases} J_{kl} e \tau_s^{-1} t\, e^{-t/\tau_s} & t>0 \\ 0 & \text{else} \end{cases}$
Synaptic weights	$J_{kl} = \begin{cases} J & \text{if synapse } kl \text{ exists and is excitatory} \\ -gJ & \text{if synapse } kl \text{ exists and is inhibitory} \\ 0 & \text{else} \end{cases}$
Parameters	Excitatory synaptic weight J, relative strength g of inhibition, synaptic time constant τ_s, synaptic delay d

D	**Spike-train analysis**
Spike trains	$s_k(t) = \sum_{j_k} \delta(t-t_{j_k})$
Population averaged firing rate	$\nu(t) = \langle s_k(t) \rangle_{k,\Delta t}$
Standard deviation of average firing rate	$\sigma = \left\langle \sqrt{\nu(t)^2 - \langle \nu(t) \rangle_t^2} \right\rangle_t$

Table 1: Description of the model and the spike-train analysis.

Self-sustained activity

SSA has been observed in networks of integrate-and-fire neurons with current-based synapses (Gewaltig, 2009). The lifetime of the activity increases sharply in a narrow band of parameter values.

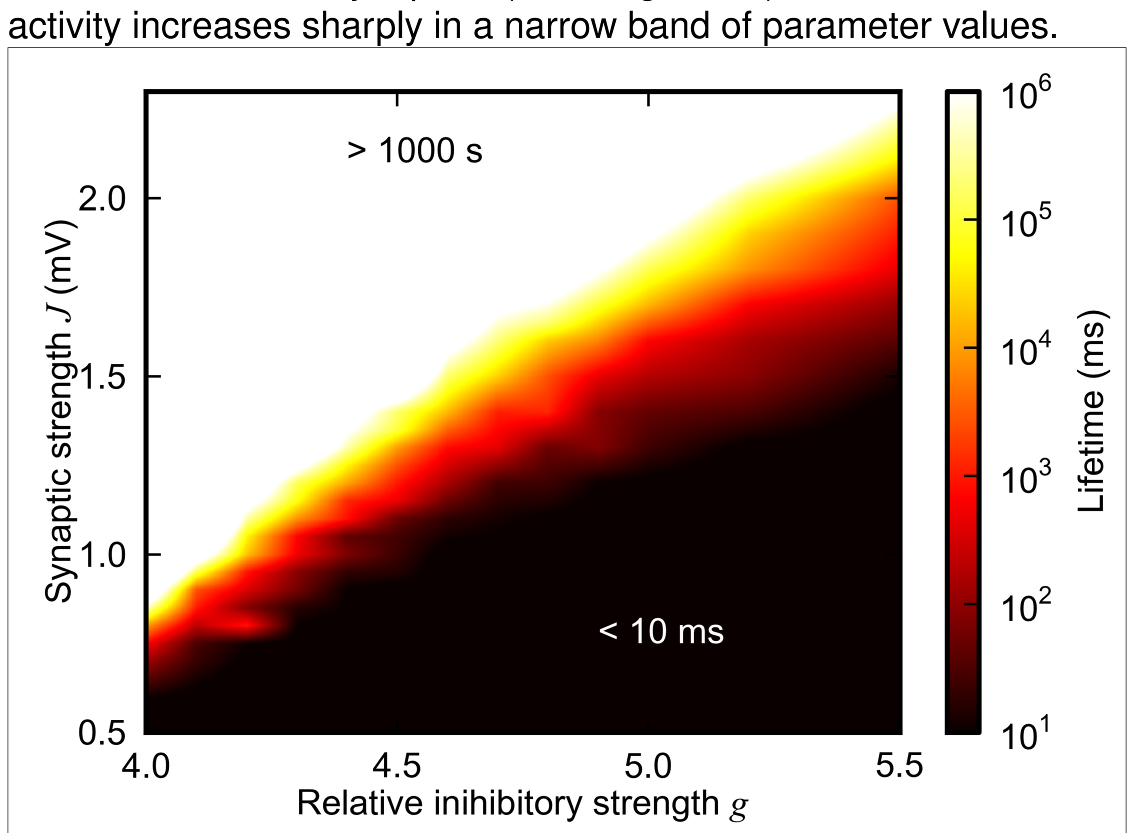


Figure 1: Dependence of the lifetime of SSA (color coded) on the level g of inhibition and the synaptic weight J in a random network of 100000 excitatory and 25000 inhibitory IaF neurons with 1% connectivity (simulation results).

Rate response function

Although the diffusion approximation is only valid in the weak synapse limit, we assume that at least in some range of strong synapses the approximation will still give quantitatively correct results. The stationary rate response function (Siegert, 1951) predicts three stationary states, a quiet state Q, an intermediate (unstable) state I and a state H with self-sustained activity for strong weights.

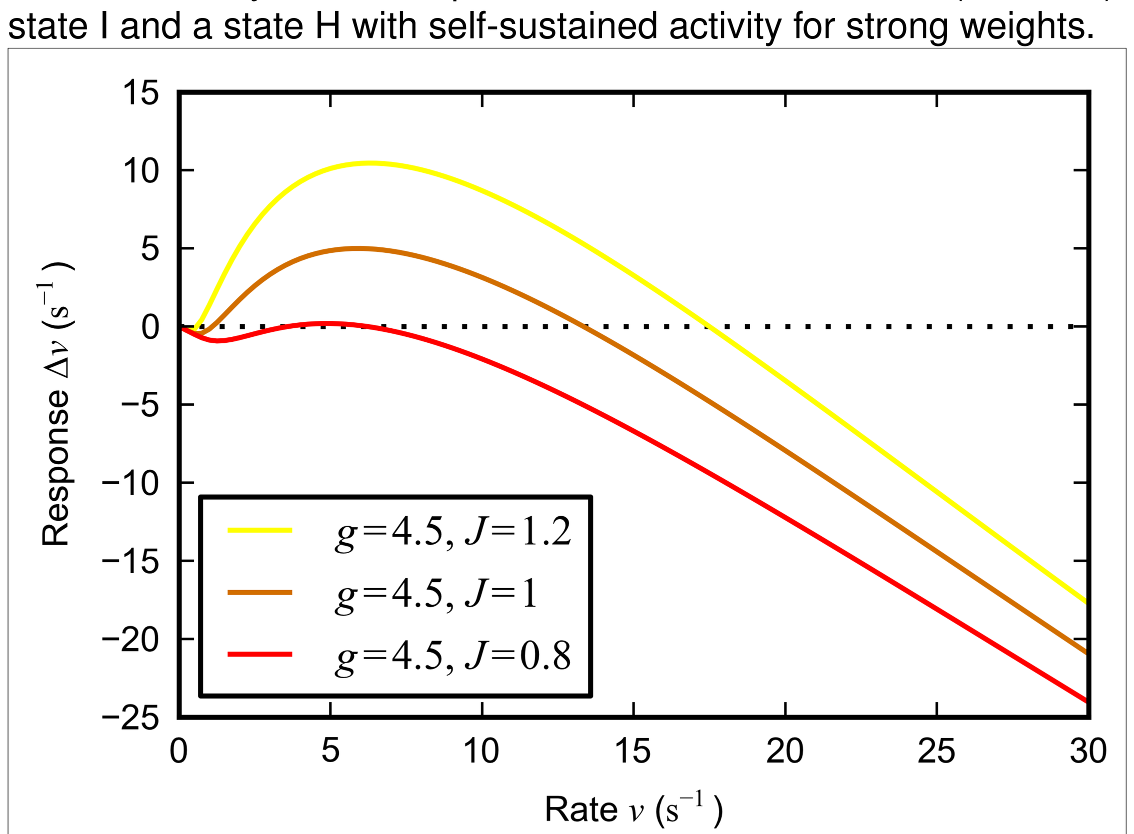


Figure 2: Rate response ($\Delta\nu = \nu_{out} - \nu_{in}$) from Siegert (1951) formula.

Stability

Linear stability analysis (Brunel, 2000) shows that the stationary H state is unstable in the strong synapse regime, where very high frequency oscillations appear. In simulations, the SSA displays significantly stronger fluctuations than externally driven activity.

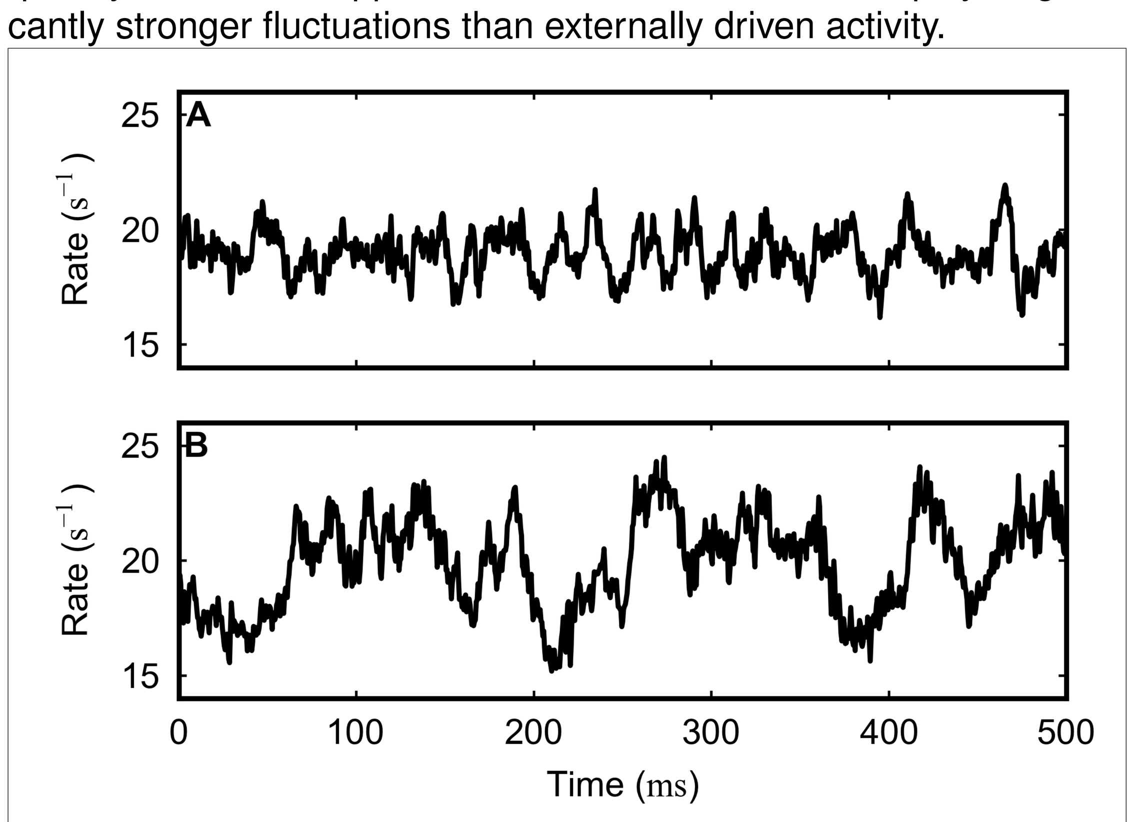


Figure 3: Network activity for **A**: externally driven weak synapses ($J = 0.1$mV); **B**: self-sustained strong synapses ($J = 1.0$mV)

Acknowledgements

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nest:: simulated()

Lifetime

We model the ultra-high frequency oscillations as stochastic fluctuations in the rate. Assuming that the rate fluctuates around the unstable H state with a Gaussian probability distribution, we calculate the probability that the rate fluctuates into the basin of attraction for the quiet Q state.

$$P_Q \propto \frac{1}{2}(1-\mathrm{erf}(x)), \quad x = \frac{\rho-\lambda}{\sqrt{2}\sigma},$$

where ρ and λ are the rates of the H and I states and σ is the standard deviation of the fluctuations. The lifetime of the SSA is inversely proportional to P_Q. σ depends on the correlations in the network (Kriener et al., 2008), which in turn depend on the firing rate (De la Rocha et al., 2007). For the present results, we have assumed that the correlation transfer is proportional to the rate.

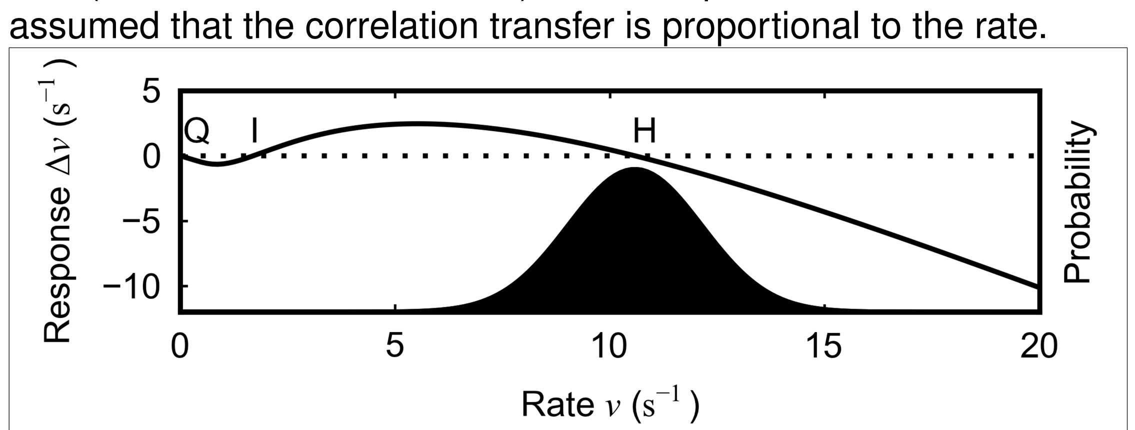

Figure 4: Stochastic model of network dynamics. Solid line: response function (see fig. 2). States Q,I,H have self-consistent rates. Filled: probability distribution for fluctuating network rate.

Comparison of model and simulations

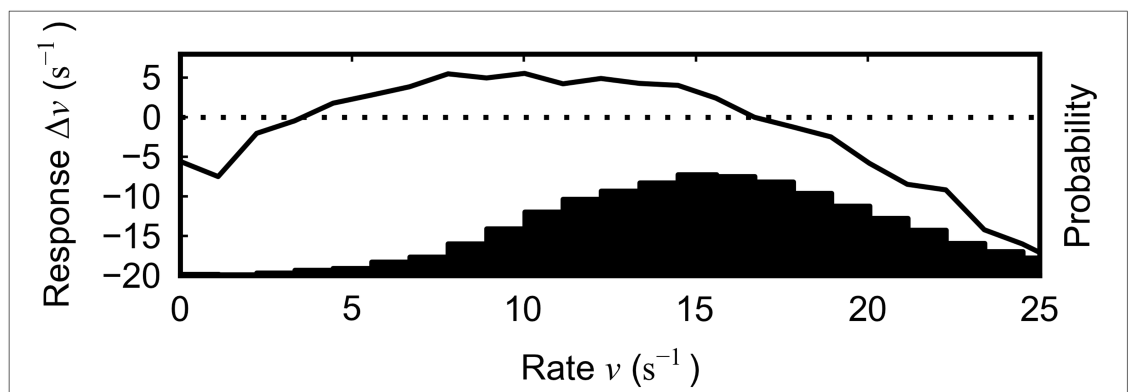

Figure 5: Result of network simulation. Solid line: measured response function. Filled: probability distribution for fluctuating network rate.

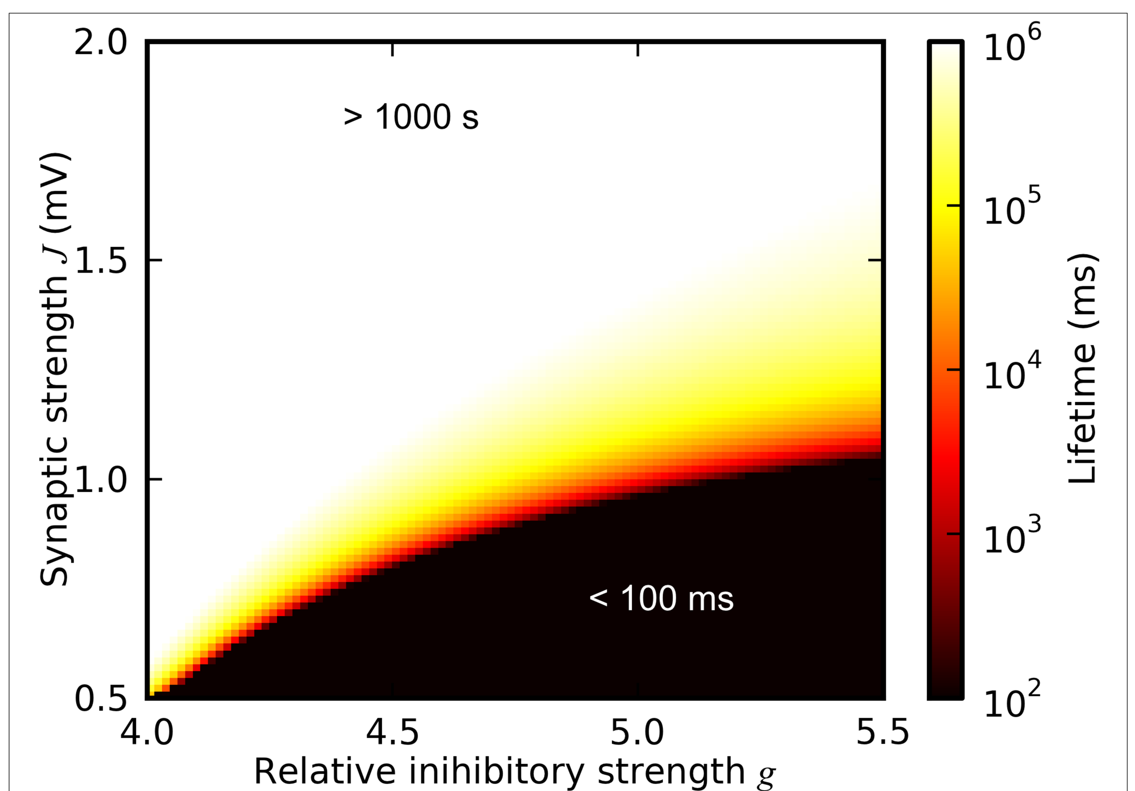

Figure 6: Model prediction for lifetime of SSA.

Fig. 6 shows the prediction from the model, to be compared to the simulation results in fig. 1. Since the diffusion approximation is expected to break down at strong synaptic weights, the model can not be expected to produce quantitatively correct results. The qualitative features of the lifetime obtained from our simplified model are however in agreement with the simulation results.

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